BERGISCHE UNIVERSITÄT WUPPERTAL

Summer term 2023

TECHNISCHE UNIVERSITÄT DRESDEN

2. Exercise Sheet in

Ordered Banach Spaces and Positive Operators

For the exercise classes on April 18 and 19, 2023

Exercise 1 (Extreme rays).

(a) Determine the extreme rays of the standard cone in ℓ^p for $p \in [1, \infty]$.

(b) Determine the extreme rays of the standard cone in $L^p([0,1])$ for $p \in [1, \infty]$.

Exercise 2 (Cones in $\mathbb{R}^d$).

(a) Let E_+ be a closed and generating cone in $E \coloneqq \mathbb{R}^2$. Show that the ordered vector space (E, E_+) is isomorphic to $\mathbb{R}^2$ with the standard cone.

(b) Give an example of a closed and generating cone E_+ in $E \coloneqq \mathbb{R}^3$ such that the ordered vector space (E, E_+) is not isomorphic to $\mathbb{R}^3$ with the standard cone.

(c) Endow $\mathbb{R}^2$ with the standard cone and let $x \in \mathbb{R}^2_+$. Does $[0,1]\, x = [0, x]$ hold?

Exercise 3 (Masquerade of cones). Show that the following ordered vector spaces are isomorphic:

(1) The space $\mathbb{R}^3$ with the ice cream cone.

(2) The space of all symmetric real 2×2-matrices with the Loewner order.

(3) The span of the three real-valued functions $\mathbb{1}$, Re, Im on $\mathbb{T}$ with the pointwise order. Here, $\mathbb{T} \coloneqq \{z \in \mathbb{C} \mid |z| = 1\}$ denotes the complex unit circle.

(4) The span of the functions $\mathbb{1}$, cos, sin on $[0, 2\pi]$ with the pointwise order.

(5) The space of all polynomial functions $\mathbb{R} \to \mathbb{R}$ of degree at most 2 with the pointwise order.

Exercise 4 (Closed faces of the cone in function spaces).

(a) Determine all closed faces of the standard cone in $L^p(\mathbb{R})$ for $p \in [1, \infty)$.

(b) Determine all closed faces of the standard cone in $C([0,1])$.

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