Book of Abstracts

Workshop on Ordered Vector Spaces and Positive Operators

29 March – 1 April 2023

Wuppertal

Patrizio Bifulco
(FernUniversität in Hagen)

Comparing the spectrum of Schrödinger operators on quantum graphs

We study Schrödinger operators on compact finite metric graphs subject to δ-coupling and standard boundary conditions often known as *Kirchoff-Neumann vertex conditions*. We compare the n-th eigenvalues of those self-adjoint realizations and derive an asymptotic result for the mean value of deviations. By doing this, we generalize recent results from Rudnick et al. and Band et al. obtained for domains in $\mathbb{R}^2$ to the setting of quantum graphs. We start this talk by introducing the basic notion of a quantum graph and discuss basic properties of heat kernels on those graphs afterwards. In this way, we are able to discuss a so-called *local Weyl law* for general vertex conditions.

[1] Patrizio Bifulco, Joachim Kerner. *Comparing the spectrum of Schrödinger operators on quantum graphs*, arXiv:2212.13954, 2022.

[2] Delio Mugnolo. *What is actually a metric graph?*, arXiv:1912.07549, 2019.

Noa Bihlmaier
(University of Tübingen)

Köhler-Semigroups and the Cyclicity Problem

In this talk we present the so-called Köhler semigroup-compactification of a power-bounded linear operator on a Banach space. We discuss its close relation to the Stone-Čech-compactification of the natural numbers. Then we use the general theory of compact right topological semigroups to find structured decompositions of the underlying operator and Banach space. These concepts yield an elegant proof that every power-bounded positive operator on a Banach lattice has cyclic peripheral spectrum.

Florian Boisen

(Technical University Dresden)

Duality of Riesz* Homomorphisms and Interval Preserving Operators in Ordered Vector Spaces

In the theory of vector lattices, there is classical duality of Riesz homomorphisms and interval preserving operators via order adjoints. We will present first results of an attempt to generalize this duality to the setting of pre-Riesz spaces and, in some special cases, even to the general setting of ordered vector spaces. Additionally, we will link this duality to an open problem concerning the characterization of Riesz* homomorphisms and present partial results.

Chun Ding

(University of Leiden)

Direct limits in categories of normed vector lattices and Banach lattices

After collecting a number of results on interval and almost interval preserving linear maps and vector lattice homomorphisms, we show that direct systems in various categories of normed vector lattices and Banach lattices have direct limits which coincide with their direct limits in natural supercategories. For those categories where the general constructions do not work to establish the existence of general direct limits, we describe the basic structure of those direct limits that *do* exist.
A direct system in the category of Banach lattices and contractive almost interval preserving vector lattice homomorphisms has a direct limit. When the Banach lattices in the system all have order continuous norms, then so does the Banach lattice in a direct limit. This is used to show that a Banach function space over a locally compact Hausdorff space has an order continuous norm when the topologies on all compact subsets are metrisable and (the images of) the continuous compactly supported functions are dense.

Roman Drnovšek

(University of Ljbuljana)

The Huijsmans–de Pagter problem in ordered Banach algebras

We will deal with a question posed by C. B. Huijsmans and B. de Pagter. Specifically, considering a positive operator T on a complex Banach lattice with its spectrum equal to $\{1\}$, is it true that T is greater than or equal to the identity operator? We will give an example of a positive element a in some ordered Banach algebra $\mathcal{A}$ such that its spectrum is equal to $\{1\}$ and it is not greater than or equal to the unit element of $\mathcal{A}$.

Patrick Hermle
(University of Wuppertal)

Uniformly ergodic semigroup representations on Banach lattices

We generalize uniform ergodic theorems to representations of semigroups. To do so we introduce a unitary spectrum for representations of semigroups. An application of this spectral theory is a Niiro-Sawasima-type theorem for a positive power bounded semigroup-representation. This a joint work with Jochen Glück and Henrik Kreidler.

Julian Hölz
(University of Wuppertal)

Uniform Almost Periodicity of Banach Lattice Homomorphisms

Each continuous mapping $\varphi : X \to X$ on a compact Hausdorff space X induces a lattice homomorphism

$$T_\varphi : C(X) \to C(X), \quad f \mapsto f \circ \varphi,$$

on the Banach lattice $C(X)$ of continuous functions $f : X \to \mathbb{C}$, called the *Koopman operator*. The ergodic properties that we examine are *uniform mean ergodicity* and *uniform almost periodicity*. We study these properties by examining the spectrum of T_φ. We present our results in the language of general Banach lattice homomorphisms, which we then apply to T_φ.

Marcel de Jeu
(University of Leiden)

Free objects in algebraic and analytic categories

In this expository lecture, we will explain how results from universal algebra can be used to construct free objects in various algebraic categories, such as the free unital vector lattice algebra over a set and over a vector space. Starting from these free objects of an algebraic nature, one can then proceed to construct normed objects, such as unital Banach lattice algebras, that solve universal problems in analytic categories. These, in turn, can be used to construct free objects in larger analytic categories, such as the free complete locally convex-solid topological vector lattice algebra over a Banach space. This analytic part is joint work with Walt van Amstel. The talk will focus mainly on the algebraic constructions.

Xingni Jiang
(Affiliation)

Positive dilation in order integrals

Let H be a Hilbert space, $B(H)$ be the Banach algebra of all bounded operators on H. Let X be a compact Hausdorff space, the Borel σ-algebra on X is denoted by Ω. Naimark's Dialtion theorem says, every positive $B(H)$-valued measure $\mu\colon \Omega \to B(H)$ can be dilated to a spectral measure P on a larger Hilbert space. Naimark's dilation theorem can be viewed as a consequence of Stinespring's factorization theorem which is a result from operator theory that any completely positive map T from a C^*-algebra A into $B(H)$ can be dilated to a $*$-homomorphism $\pi\colon A \to B(K)$, where K is a Hilbert space. However, not all of the operator-valued measure has a Hilbert dilation space, so does operators. Thus, it is necessary to develop a dilation theory to include non-Hilbertian space. [1] obtains dilation results in Banach spaces, and Banach algebras.
In [2], we develop a comprehensive theory of partial order vector-valued measures and order integrals. In [3] and [4], we discuss the related positive representation theorems. In this talk, we will give a brief introduction of the measure theory in partially ordered vector space and algebras, and then we apply the elementary dilation construction into it. In the end, we will discuss the close relationship between the dilation of partial order vector-valued measures and the dilation of regular operators generated by order integrals. This is joint work with Rui Liu .

References

1 B. Liu D.Han, D. R. Larson and R.Liu. Operator-Valued Measures, Dilations, and the Theory of Frames, Memoirs of the American Mathematical Society. American Mathematical Society, (2014), volume 229.

2 M.de Jeu, X. Jiang. Order integrals, Positivity (2022) 26:32.

3 M.de Jeu, X. Jiang. Riesz representation theorems for positive linear operators, Banach Journal of Mathematical Analysis (2022)16:44.

4 M.de Jeu, X. Jiang. Riesz representation theorems for positive algebra homomorphisms, arXiv:2109.10690.

Marko Kandić
(University of Ljubljana)

Prime ideals in vector lattices

In this talk, we consider the set of all prime ideals in vector lattices and how the properties of the prime ideals structure the vector lattice in question. The different properties that will be considered are, first, that there are only finitely many prime ideals; secondly, that every prime ideal is principal; and lastly, that every ascending chain of prime ideals is stationary.

Sebastian Kaul

(Technical University Dresden)

Criteria for positive spectral radii on Banach lattices and partially ordered Banach spaces

It is well known that if a compact positive operator $T\colon X \to X$ on a partially ordered Banach space (X, K) has a strictly positive spectral radius $r(T)$, then $r(T)$ is an eigenvalue of T and the corresponding eigenvector is an element of the cone K. The Theorem of de Pagter gives a condition when such an operator has a strictly positive spectral radius in the case of Banach lattices. In the talk we will present this theorem and a sketch of its proof. In order to find a similar result like the one of de Pagter in the setting of partially ordered Banach spaces, we then show how ideals are defined on partially ordered vector spaces and present new results in regards to representations of different kinds of specific ideals, especially for the null ideal. In the end we give some ideas which different properties one could use to show the strict positivity of the spectral radius of a compact positive operator on a partially ordered Banach space. Different topics are joint work with Anke Kalauch, Janko Stennder and Jochen Glück.

Elena Klimova

(Goursat-type nonlocal problem for a fourth-order loaded equation)

Dresden University of Applied Sciences (HTW Dresden)

Initial and boundary value problems for partial differential equations are well studied. However, it has recently become evident that various processes and phenomena of modern natural science lead to non-classical problems for differential equations. In this presentation we will consider nonlocal boundary value problems, i.e. problems where conditions are given not only on the boundary of a domain, but also at some other points inside the domain. They belong to the class of nonclassical problems.
We turn our attention to some problems with nonlocal integral conditions for a loaded equation. Most of the papers dealing with boundary value problems with integral conditions deal with second order equations. In this presentation we will consider a non-local problem with integral conditions for a loaded fourth order equation. Factorization allows to transform this problem to two problems for second order equations: the Goursat problem for an integrodifferential equation and an integral analogue of the Goursat problem for a simple hyperbolic equation. The single solvability of the problem in question is proved.

Marjeta Kramar Fijavž
(University of Ljubljana)

Relatively uniform spectral theory for operators on vector lattices

A net $(x_\alpha)_\alpha$ in a vector lattice X *converges relatively uniformly* to $x \in X$, if there exists $u \in X$ such that for each $\varepsilon > 0$ there exists α_0 such that

$$|x_\alpha - x| \leq \varepsilon \cdot u \quad \text{holds for all } \alpha \geq \alpha_0.$$

We introduce the notions of spectrum and spectral radius of a linear operator on X in terms of the relative uniform convergence. For this notions we can obtain several properties that resemble the properties of the usual spectrum on a Banach space. We are especially interested in tools needed for the further development of the theory of relative uniform continuous operator semigroups studied in [1, 2, 3].

Literatur

[1] J. Glück M. Kaplin. Order boundedness and order continuity properties of positive operator semigroups. arxiv.org/abs/2212.00076, preprint 2022.

[2] M. Kandić and M. Kaplin. Relatively uniformly continuous semigroups on vector lattices. *J. Math. Anal. Appl.*, 489(1):124139, 24, 2020.

[3] M. Kaplin and M. Kramar Fijavž. Generation of relatively uniformly continuous semigroups on vector lattices. *Anal. Math.*, 46:293–322, 2020.

Kieran Power
(University of Kent)

Compactification of symmetric cones under the Hilbert distance

The horofunction compactification of a metric space is a well known concept that goes back to Gromov, with many applications in dynamics, geometry, and complex variables. In general the horofunction compactification is hard to compute. In certain classes of finite dimensional normed spaces a duality phenomenon has been observed.
In work with Bas Lemmens we have shown that this duality phenomenon occurs for a variety of classes of Finsler metric spaces. In this talk we will focus specically on symmetric cones under the Hilbert distance, where we exploit the interplay between the Hilbert metric and order structure of the cone, to explicitly construct the horofunction compactification for these spaces. We begin with a brief discussion of hyperbolic models of geometry and the compactification of classical symmetric spaces. We then introduce the machinery needed to show an explicit homeomorphism from the horoboundary onto the closed unit ball of the dual of the tangent space at the base point.

Agnes Radl

(University of Applied Sciences Fulda)

Embeddability of positive matrices

It is a well-known problem in probability theory whether a Markov matrix is embeddable into a Markov semigroup. Even today it is an active field of research, see e. g. the recent survey "Notes on Markov embedding" by M. Baake and J. Sumner [1]. We consider a related problem: Given a (finite or infinite) matrix T, is it embeddable into a real/positive C_0-semigroup, i. e., is there a real/positive C_0-semigroup $(T(t))_{t\geq 0}$ such that $T(1) = T$? We will give necessary and sufficient conditions for embeddability of a real matrix into a real C_0-semigroup. In the case that T is positive we will present necessary conditions for embeddability of T into a positive C_0-semigroup. Moreover, we will give a full description in the case that T is a 2×2 matrix.
This presentation is based on a joint work with T. Eisner [2].

[1] M. Baake and J. Sumner, *Notes on Markov embedding*, Linear Algebra Appl. **594** (2020), 262–299.

[2] T. Eisner and A. Radl, *Embeddability of real and positive operators*, to appear in Linear and Multilinear Algebra.

Janko Stennder

(Technical University Dresden)

Algebraic structures in pre-Riesz spaces

We consider pre-Riesz spaces together with associative multiplications given by different kinds of bilinear maps. The aim is to generalize results from the theory of lattice-ordered algebras, in particular, f-algebras, almost f-algebras, and d-algebras. Based on this, we study properties of generalizations of bi-orthomorphisms, bi-Riesz homomorphisms, and orthosymmetric maps on pre-Riesz spaces. Especially on order dense subspaces of $\mathrm{C}(K)$-spaces, we established some first results. Among other things, statements regarding algebra homomorphisms, nilpotent elements, and representations are presented.

Cormac Walsh

(INRIA, Palaiseau)

Order and metric structures on cones

In an order-unit space, there are a couple of very interesting metrics on interior of the positive cone that are defined naturally in terms of the order structure. The properties exhibited by these Hilbert and the Thompson metrics, as they are known, depend to a great extent the geometry of the cone. In this talk, I will survey what is known about them, convering such topics as their isometry groups and their horofunction boundaries.

Martin Weber
(Technical University Dresden)

Cesàro Sequence Vector Lattices. Duals and Ideals of Finite Elements

The Cesàro sequence spaces $\mathrm{ces}_0, \mathrm{ces}_p$ (for $1 < p < \infty$) and ces_∞, defined as

$$\mathrm{ces}_p = \{x \in \mathbb{R}^{\mathbb{N}} \colon C\,|x| \in \ell_p\}$$

turn out to be real vector lattices, and with respect to a corresponding (naturally introduced) norm they are Banach lattices, and so possess (or not possess) some interesting properties. We study the relations to their generating ideals c_0, ℓ_p and ℓ_∞, describe the finite elements in these lattices and in their dual spaces for $p \in \{0\} \cup [1, \infty)$. Recall that a detailed analysis of continuous functions with compact support (i.e. finite functions) leads to the notion of finite, totally finite and self-majorizing elements in an arbitrary Archimedean vector lattice E, e.g. an element φ of a vector lattice E is called *finite* if there exists an element $z \in E$ (a majorant of φ) satisfying the property: for each $x \in E$ there is a number $c_x > 0$ such that the inequality

$$\boxed{|\boldsymbol{x}| \wedge \boldsymbol{n}|\boldsymbol{\varphi}| \leq \boldsymbol{c_x}\, \boldsymbol{z}}$$

holds for any $n \in \mathbb{N}$.
Finally, after the Cesàro sum $\mathrm{ces}_p(\mathfrak{X})$ of a sequence of Banach spaces is introduced, we characterize their duals and the finite elements in these sums if the summed up spaces are Banach lattices. This is done by means of a remarkable extension of the corresponding result for direct sums.