

# Splitting Approaches for Fokker-Planck Equations: Theory and Applications

J. Geiser  
Ruhr University of Bochum  
Department of Electrical Engineering  
and Information Technology  
Universitätsstrasse 150  
D-44801 Bochum, Germany  
[juergen.geiser@ruhr-uni-bochum.de](mailto:juergen.geiser@ruhr-uni-bochum.de)

We are motivated to solve the Fokker-Planck (FP) equation, which is a partial differential equation in statistical mechanics. The FP equations are applied in plasma physics, e.g., particle collision terms in the well-known Boltzmann equation, see [1], or fluid dynamics, e.g., thin film formation and growth, see [2]. Further, the solution of the FP equation can be written as a stochastic process with the Langevin equation.

Mathematically, we are interested on solving the FP equations with respect to splitting approaches, which allows to accelerate the solvers, see [3].

We discuss the two possible splitting approaches:

- FP-splitting: We apply the operator splitting of higher order parabolic equations, see [4].
- Langevin-Splitting: We apply the operator splitting of stochastic processes, see [5].

The numerical analysis of the splitting approaches with respect to error estimates are presented. Further, we discuss possible flexibilisations to control problems of stochastic processes, see [6]. Numerical experiments of a plasma and thin-film models are discussed.

At the end of the talk, we summarize our results.

## References

- (1) J. Geiser. Multiscale Modelling and Splitting Approaches for Fluids composed of Coulomb-interacting Particles. Special-Issue: Problems with Multiple Time-scales Mathematical, Journal: Mathematical and Computer Modelling of Dynamical Systems, edited by J.Geiser, Taylor and Francis, Abingdon, UK, accepted January 2018.
- (2) J. Geiser. Operator Splitting Approaches of Deposition of Brownian Particles based on Fokker-Planck Equation. AIP Conference Proceedings Paper, ICNAAM 2018 (13.-18. September, 2018), Rhodes, Greece, accepted August 2018.
- (3) J. Geiser. Iterative Splitting Methods for Differential Equations, Chapman & Hall/CRC Numerical Analysis and Scientific Computing Series, edited by Magoules and Lai, 2011.
- (4) S. Ganesan and L. Tobiska. Operator-splitting finite element algorithms for computations of high-dimensional parabolic problems. Applied Mathematics and Computation, 219: 6182-6196, 2013.
- (5) J. Geiser. Iterative Semi-implicit Splitting Methods for Stochastic Chemical Kinetics. Lecture Notes in Computer Sciences, Proceeding of the Seventh Conference on Finite Difference Methods: Theory and Applications, June 11-16, 2018 Lozenetz, Bulgaria, accepted June 2018.
- (6) M. Annunziato and A. Borzi. A Fokker-Planck control framework for multidimensional stochastic processes, Journal of Computational and Applied Mathematics, 237: 487-507, 2013.