INFINITE-DIMENSIONAL SYSTEMS, STATE- AND FREQUENCY DOMAIN TECHNIQUES.

HANS ZWART

Outline of the course

In this course we will study evolution equations described by partial differential equations. Many of these partial differential equations model physical behaviours, and hence it is only natural to take this into account. Therefore, we write our partial differential equation as a port-Hamiltonian system. The Hamiltonian often equals the energy of the system. For this class we identify those boundary conditions for which the (homogeneous) evolution equation is well-posed, i.e., possesses a unique solution for every initial condition with finite energy.

By controlling some of the boundary conditions and observing others, we obtain a boundary control system with observations. By a proper choice of these boundary control and observation, we can have the following relation between the (internal) energy of the system, $E(t)$, and the in- and output, $u(t)$ and $y(t)$, respectively,

$$\dot{E}(t) = u(t)^T y(t). \tag{1}$$

Like any linear, time-invariant system a port-Hamiltonian system possesses a transfer function, $G(s)$. Although this function can be hard/impossible to calculate, using an energy/power estimate like (1), properties of this transfer function can be derived without any difficulty. For instance if (1) hold, then G is positive real, i.e.,

$$G(s) + G(s)^* \geq 0, \qquad \text{for } \operatorname{Re}(s) > 0$$

Equation (1) indicates clearly how the system can be stabilised. Namely, by choosing $u(t) = -ky(t)$ the energy decays, and so stability may be expected. Under proper additional conditions, we show that this indeed holds. We show that the same conclusion can be drawn when applying a non-linear controller with damping. For linear control, we show also how stability can be proved using the transfer function, i.e., using frequency domain techniques. Summarising, we address the following main topics:

- Existence of solutions for linear, homogeneous and inhomogeneous port-Hamiltonian systems.
- Transfer functions.
- Stabilisation of port-Hamiltonian systems, time and frequency domain techniques, linear and non-linear control.